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Intensionality and Context Change

Towards a Dynamic Theory of Propositions and Properties

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Abstract. It is arguably desirable to have a theory of meaning that (i) does not identify propositions with sets of worlds, (ii) enables to capture the dynamic character of semantic interpretation and (iii) provides the basis for a semantic program that incorporates and extends the achievements of Montague semantics. A theory of properties and propositions that meets these desiderata is developed and several applications to the semantic analysis of natural languages are explored.

Key words: properties, propositions, dynamic semantics, Montague semantics, possible worlds, characters, λ-calculus, Scott Domains, Frege Structures.

1. Goals and Motivation

In trying to overcome some of the limits inherent in Montague's seminal work, much recent research in natural language semantics has focussed on two main questions, having to do with the "granularity" of meaning and the "dynamics" of interpretation.

The granularity question manifests itself primarily in attempts to arrive at a sufficiently fine grained characterization of information bearing structures. In particular, in possible worlds semantics propositions are modelled as sets of worlds. While this works well for many purposes, it also appears to lead to serious difficulties connected to the fact that any two logically equivalent propositions are identified. An alternative line of inquiry, which is being actively pursued, is to take propositions and properties (rather than possible worlds) at face value (i.e. as basic, unreduceable notions).*

There are also many attempts to view meaning not just in terms of semantic content but also more dynamically in terms of how it affects the information available to illocutionary agents. According to this view, the meaning of a sentence is not the proposition it denotes, but the way a sentence may change its context

* See, e.g., Thomason (1980), Barwise and Perry (1983) or the papers in Chierchia *et al.* (1989), among many others.

of utterance. Many recent approaches to anaphora argue that there is much to be gained by pursuing this view.**

These two lines of inquiry have been developed for the most part separately. Furthermore, they have lead to often radical revisions of Montague's framework which has meant giving up some of its good aspects. While this is a natural consequence of exploring new alternatives, one would want eventually a framework capable of incorporating fully whatever merits classical Montague semantics may have. So my goal here it to work out an approach where these two main streams of current semantic research can comfortably and profitably coexist with each other and can also coexist with what is alive of classical Montague semantics. The framework that I will develop is based on work in Property Theory (especially Aczel, 1980; Turner, 1990) and links it with dynamic theories of anaphora such as those of Groenendijk and Stokhof (1990). It turns out that the framework that emerges from this synthesis uses crucially a notion closely related to Kaplanian "characters" (cf. e.g., Kaplan, 1979).

Given these general goals, let me begin by fleshing out what strike me as a minimal set of desiderata. I will do this first impressionistically. It seems desirable to have a framework with the following features:

(1) a) It should allow to give a semantics for at least the same range of linguistic phenomena covered in Montague's PTQ (e.g. Montague's treatment of NPs as generalized quantifiers, Montague's treatment of transitive verbs, etc.)

b) It should be hyperintensional: propositions are not to be assimilated to sets of worlds.

c) It should be reflexive: properties are to be allowed to be predicated of themselves.

d) It should be dynamic: aspects of the dynamic unfolding of discourse are to be naturally representable in it.

In the rest of the present introduction, I'd like to elaborate a bit on each of these points.

1.1. Reformists and Revolutionaries

Most of the semantic approaches that developed after Montague are, as Muskens (1989) puts it, revolutionary. They seek to replace Montague semantics with theories that are presented as completely new. Muskens points out that this often leads to throwing away the good with the bad and advocates instead a course of enlightened reforms.

** See for example Stalnaker (1979), Kamp (1981), Heim (1982), Groenendijk and Stokhof (1990), Chierchia (1992), among many others.

I also think, with Muskens and others, that there are several features of Montague Semantics that are worth keeping. For example, I think that one of the clearest way of understanding how syntax relates to semantics is in terms of a compositional Montague-style map from the relevant level of syntactic representation into meanings (where the latter are isolated with the help of a particular logical theory). By relevant level of syntactic representation I mean essentially surface syntactic structure enriched by a representation of scope and anaphoric links, however that may be achieved.

Another important characteristic of Montague semantics is that NP's are treated uniformly as generalized quantifiers, where the latter are viewed as sets of properties. In Montague's Intensional Logic (IL), the meaning of *a man* and *every woman* is analyzed as:

(2) a) a man' = $\lambda P \exists x$ [man'$(x) \wedge {}^{\vee}P(x)$]

b) every woman' = $\lambda P \forall x$ [woman'$(x) \rightarrow {}^{\vee}P(x)$].*

This approach has lead to the discovery of many interesting properties of NP-meanings and allows for a simple treatment of coordinated structures like:

(3) [A man and every woman] left.

Here two NP's are conjoined. The meaning of *and* in this context can be taken to be a simple pointwise generalization of sentential *and*, specified along the following lines:

(4) $\mathrm{NP}_1' \wedge \mathrm{NP}_2' = \lambda P\, [\mathrm{NP}_1'(P) \wedge \mathrm{NP}_2'(P)]$.

This provides the basis for a simple and elegant treatment of Boolean operators in natural language across different syntactic categories (for details, see Partee and Rooth, 1983; Keenan and Faltz, 1985).

As a further example, consider Montague's treatment of transitive verbs. They are uniformly treated as functions from NP-denotations into predicates. Extensional verbs are further constrained as follows:

(5) kiss' = $\lambda \delta \lambda x \delta({}^{\wedge}\lambda y[\mathrm{kiss}_*(y)(x)])$
where δ is a variable ranging over NP-meanings and kiss_* is a first order two place relation (of type $< e, < e, t >>$).

As a consequence, (6a) will be equivalent to (6b), but (7a) will not be equivalent with or entail (7b):

(6) a) John kisses a friend $\Longrightarrow$ kiss'(a friend')(j)

b) $\exists x$ [friend'$(x) \wedge \mathrm{kiss}_*(x)(j)$]

* I adopt the convention that for any expression α, α' is the translation of α into the logic used to do semantics (in Montague's case, IL). I will refer to α' as to "the logical form of α".

(7) a) John needs a friend $\Longrightarrow$ need'(a friend')(j)

b) $\exists x$ [friend'(x) $\wedge$ need$_*(x)(j)$].

While this many not be the definitive word on intensional verbs, it works remarkably well. It characterizes in an elegant way the pattern of entailments that intensional verbs give raise to.**

This is a sample of proposals by Montague that while not problem-free appear to be still rather cogent and at any rate superior to alternatives currently available. On most of the postmontagovian semantic programs (such as Discourse Representation Theory (DRT) or Situation Semantics) it is not obvious how they are going to be preserved. One of our goals will be to develop a theory where these aspects of Montague Semantics can be directly incorporated.

1.2. HYPERINTENSIONALITY

As mentioned at the outset, the thesis that propositions are sets of worlds fixes very rigidly their identity criteria. Two propositions are identical iff they have the same truth-conditions (or, equivalently for our present purposes, iff they are logically/necessarily equivalent). Many reasons of dissatisfaction with this have been voiced in the literature. Some have to do with the very notion of possible world. Such a notion, it can be and has been argued, carries along obscurity and a wealth of problematic metaphysical assumptions. If one is doubtful about possible worlds, then it seems sensible to explore alternative ways of talking about propositions or, more generally, information bearing structures. From this perspective, taking the notion of proposition as basic while exploring what structural features we might take it to have looks like a prima facie plausible strategy. The plausibility of this strategy is enhanced by the rather minimal ontological assumptions it carries along. For all we know, the best theory of propositions might well turn out to be compatible with, say, a nominalistic view of what propositions are.

Things are different, however, if we do believe that possible worlds are an essential component of the proper understanding of mental attitudes. Suppose we want to maintain that mental attitudes are relations of rational agents to sets of worlds and that propositions are primarily used to discriminate among alternative states of affairs. Is it then plausible to regard propositions as anything but sets of such states? The answer is not obvious. True, this view of mental attitudes and propositions leads into difficulties when it comes to analyzing mathematical beliefs. But bringing in new primitives surely does not per se make problems disappear. Hence, it is not clear it constitutes the right move.

In the rest of this subsection, I'd like to give an indication of why viewing propositions as primitives might be of use even from the point of view of a possible

** For a recent alternative to Montague's theory of intensional verbs, cf. Zimmerman (1993). Zimmerman's approach has as consequence that intensional verbs with universally quantified object such as "I want every policeman in this city" lack an opaque reading. This claim is not supported by my intuitions.

world theory of mental attitudes. I have no knock down argument that it must be so, also because there are several ways in which a possible world theorist might be skeptical about propositions as primitives. But we can try to flesh out one plausible set of reasons for such skepticism and then argue that they are not as solid as they might prima facie seem. While this is certainly not going to be conclusive, it does lend further plausibility to the pursuit of an autonomous grounding of the notions of property or proposition.

It is interesting, in this connection, to consider a point made by Stalnaker (1984), from the point of view of what he calls "the pragmatic picture of intentionality", a picture I find congenial. Stalnaker's point, as I understand it, can be informally stated in its barest outline along the following lines. As rational agents, we are confronted with alternative possible outcomes to our actions and we have attitudes towards such alternatives: we want to bring some of them about and prevent others from taking place. Thus, mental attitudes are attitudes towards alternative states of affairs. This justifies, for example, analyzing belief as a relation $\mathrm{R}_{\mathrm{believe}}$ between rational agents and possible worlds. Roughly speaking, we can say that u bears the $\mathrm{R}_{\mathrm{believe}}$ relation to w iff u is disposed to act in the pursuit of his desires as if he was in w. Propositions are ways of discriminating among states of affairs. This makes them very useful in characterizing our belief-states. In particular, we can say that u believes that p iff in all of u's belief worlds (i.e., u's doxastic alternatives), p is the case. More explicitly:

(8) believe $(u, p) = \forall w\ (\mathrm{R}_{\mathrm{believe}}(u, w) \rightarrow T(w, p))$
where $T(w, p)$ is to be read as "p is true in w".

But under this view it is clear that if p and q have the same truth-conditions and one believes p then one believes also q. This is as it should, given that all that matters are the states of affairs that p and q respectively pick out. In other words under the view of belief as a disposition to act causally tied to our environment, we would want (at least when not focussing on mathematics) precisely this consequence. Stalnaker concludes that "this implies that the thesis that necessarily equivalent propositions are identical – the main substantive consequence of the possible worlds analysis of propositional content – is a thesis that is tied to and motivated by the causal-pragmatic explanation of intentionality" (Stalnaker, 1984, p. 24). That is, if one buys the pragmatic picture of mental states, one should also be ready to accept that propositions are no more than sets of worlds.

However, I fail to see how Stalnaker's conclusion follows from the argument he gives. For suppose that we take the notion of proposition in the definition (8) as primitive. What would change in the pragmatic picture of intentionality? As far as I can see, nothing whatsoever. If two propositions p and q are true in the same worlds and to believe p is for p to be true in one's doxastic alternatives, then one can believe p iff one believes q. This remains so whatever propositions may be. It directly follows from (8) as such. But then, the thesis that necessarily equivalent propositions are identical cannot be so tightly linked to the pragmatic picture of mental states as Stalnaker seems to suggest.

There might be other reasons for being weary of taking propositions as primitives. For example, if one is trying to understand what intentional states are in naturalistic terms, as Stalnaker is, one might feel that by taking propositions as primitives one reduces a notion in need of explanation (intentional state) to one equally in need of explanation (proposition or property). But propositions and properties, under the line we are exploring, do come with a set of axioms that determines their structure and enables us to test the empirical consequences of claims concerning their role in a theory of mental attitudes (or in a theory of meaning). In this regard, their status does not seem to be any different from things like numbers, sets, events or for that matter, possible worlds. If they are problematic, it is not because they slide intentionality in surreptitiously. As far as I can see, any line of argumentation with which one can defend talk of numbers, sets or possible worlds can be used with the same degree of success with propositions and properties. In fact, one can even argue that things like numbers or possible worlds really are just properties of a certain kind. This comes as no surprise for numbers. To say, for example, that those boys are no more than ten surely ascribes a property to a group, if anything does. Similarly, to say that p holds in w does prima facie look like the attribution of a property to p. Worlds as properties might be regarded as encoding patterns of consistency and maximality among propositions.

There is another angle from which one might be reluctant in accepting propositions, from the point of view of the possible worlds theory of propositional attitudes. Suppose we can explain intentional states (and the semantics of propositional attitudes verbs) just in terms of possible worlds. Then there would be no need to resort to new primitives. If we can get by with familiar set theoretic constructs out possible worlds, why should we bother with new unfamiliar propositional structures?

But can we get by with possible worlds and standard set theories? It is far from obvious that we can. Granted that we are willing to agree on something like definition (8) as our starting point, we know that (8) is lacking in a number of ways, most prominently when it comes to considering intentional states about mathematics. The analysis embodied in (8) needs to be elaborated on in some way. Its most reasonable elaborations to this date tend to bring a structural element into consideration. For example, Stalnaker argues that mathematical beliefs have a metalinguistic character having to do with the syntactic structure of mathematical sentences. Cresswell (1985) appeals to a more abstract kind of logical structure with his "structured meanings". These elaborations are not fully worked out. It is far from clear that once they are, familiar set theoretic constructs and possible worlds will suffice to make sense of them. As a matter of fact, in the case of the structured meanings approach, it is pretty clear that they will not. Such an approach seems to call for a non canonical notion of set.* But that is arguably just what properties are: non canonical sets.

Be that as it may, if we insist that the identity conditions on propositions be rigidly fixed by logical equivalence, the structural elements we need to make (8) consistent with the facts have to come from something other than and external to propositions themselves. If instead we leave open the identity criteria for propositions, we have a ready niche for the required structure. And we can form hypotheses as to what such structure might be like without having to commit ourself too soon on its relationship to a specific syntax of a specific language (much like we can study general properties of classes of programs independently of specific implementations). I am not ready here to try out a specific reformulation of (8). I am noticing that such a reformulation must bring in extra structure and that extra structure might well be propositional structure. Or at the very least, whether it is should be left open and hopefully set on empirical grounds.

As I said, none of the above considerations is conclusive. But they do lend preliminary plausibility to the claim that even from the point of view of classical possible worlds semantics it is worth exploring ways of characterizing the structure of propositions that treat their identity conditions on a par with that of ordinary individuals.

What are, in the end, propositions? Intuitively, they are quantities of information. For what is worth, I am inclined to follow Frege and Russell and think of propositions as structured objects. I don't think of them as Platonic entities, but as structures constructed out of basic regularities occurring in nature. But whether one likes this picture or not, the point is that it doesn't really matter what propositions are to the extent that they have enough logical structure to support a comprehensive semantic program and plausible view of mental attitudes.

1.3. Reflexivity

There are reasons to believe that whatever properties are, they can apply to themselves. One often quoted reason is the following. Consider the property of being selfidentical. This is a property that any x has just in case x is identical to x. But clearly the property of being selfidentical is identical to itself (what would it mean to claim the contrary?). Thus the property of being selfidentical holds of itself. Many other examples can be constructed along similar lines. They are all rather abstract, but I don't see where they go wrong.

When we were first exposed to set theory, we have been taught that sets cannot belong to themselves. Horrible things seem to happen if we assume otherwise. On the classical possible worlds analysis of properties, they are modelled as functions from individuals into sets of worlds. Functions are sets of ordered pairs. Thus properties inherit from sets the alleged incapacity for self-predication. A property cannot be predicated of itself because a set of ordered pairs cannot contain itself.

* This has to do with the treatment of iterated belief. Cf. Cresswell (1985, pp. 85 ff.) for discussion.

But today, several sensible ways of designing theories of sets and properties have been devised that are reflexive in that they allow sets to belong to themselves and properties to hold of themselves. This makes it possible to make good sense (in more than one way) of the idea that properties like "being identical to oneself" hold of themselves. We are free from the burden of trying to explain these facts away.

I also believe that there are empirical reasons for a reflexive theory of properties. Natural languages have constructions like infinitives and gerunds, exemplified in what follows:

(9) a) John likes [to play tennis]

b) John likes [playing tennis].

What are the semantic values of infinitives and gerunds (whatever their syntactic structure may be)? I think, with Montague, that there are good reasons to believe that they are properties or property-like entities. The most plausible compositional semantics of, say, (9b) ought to be roughly:

(10) like'(j, λx [play'(x,tennis)]).

(10) is understood as saying that the like-relation holds between John and the property of playing tennis. What John likes is to have that property. I will not argue for this thesis here.* I will simply assume it and point to some of its consequences.

There are several ways of implementing the hypothesis that infinitives and gerunds denote properties. One can say that, for example, *is fun* and *being fun* have the same semantic values and that their difference is purely syntactic. If this is so, then *being fun is fun* predicates the property of being fun of itself. Or one can give a Fregean slant to this view. One can regard *is fun* as an unsaturated structure, whose saturation results in a proposition. *Being fun*, instead, is an object (a saturated particular) systematically correlated to that unsaturated structure (perhaps *being fun* is a naturalistic object, like a state). Also on this second view, we need a theory of properties that supports a form of self-predication. This is so because each finite VP denotes a property and for each-such VP there is a (different) corresponding gerund or infinitive, which denotes an object that represents it. And, as we just pointed out, nothing prevents a finite VP from meaningfully applying to its own gerundive counterpart. So on the Fregean view, for each property, there must be an individual correlate and no two properties should have the same correlate. Properties can apply meaningfully and, possibly, truthfully to their own correlates. This too calls for reflexive structures.

So on either view, examples like (11a, b) appear to be cases of self-predication that, as it were, occur in nature.

(11) a) being fun is fun

* See e.g. Chierchia (1984, 1989) and references therein.

b) being crazy is crazy.

1.4. Dynamics

Uttering a sentence affects the context of utterance in many ways. One such way is by expanding the information available to the illocutionary agents. Another way is by setting up a topic that can be picked up in subsequent discourse. Let us illustrate this by looking at anaphora. Contrast (12a) with (12b).

(12) a) A man_1 I knew walked in. He_1 looked sad

b) *Every man_1 I knew walked in. He_1 looked sad.

The indefinite NP *a man* is generally analyzed as an existentially quantified structure. This structure can act as an antecedent for subsequent pronouns. Thus the existential quantifier associated with *a man* must be able, as it were, to bind beyond its syntactic scope. Impressionistically, one can say, with Kartunnen (1976), that indefinites set up "discourse referents" that can be referred to in subsequent discourse. Universally quantified terms do not have this property, as (12b) shows. The same can be seen by the well known and widely discussed contrast in (13):

(13) a) Every man who has a donkey beats it

b) *Every man who has every donkey beats it.

In (13a) too, the indefinite *a donkey* antecedes a pronoun which is not in its syntactic scope (by any reasonable definition of syntactic scope). This illustrates how the meaning of NP's seems capable of affecting the information state of an illocutionary agent by opening and shutting off "discourse referents" and the information associated with them.

This type of problems constitutes, in particular, the main motivation for one of the most successful approaches to meaning that have been developed after Montague, namely Discourse Representation Theory (DRT, cf. Kamp, 1981; Heim, 1982). Recently, an interesting approach to these issues, related to DRT, has been developed by J. Groenendijk and M. Stokhof (1990, 1991) (G&S, henceforth). G&S try to do justice to the insights that have lead to DRT but couch them within a dynamicized version of Montague's IL. Their general line is also aimed at preserving what is still usable of classical Montague semantics. In the rest of this section, I will try to give a brief summary of G&S's theory.

Let ϕ be the Montagovian translation of a sentence S into Montague's IL. We might represent the way in which ϕ is used to update the context in which it is uttered as:

(14) $\uparrow \phi = \lambda p\, [\phi \wedge {}^{\vee}p]$, where p ranges over propositions.

The function in (14) determines a set of propositions: those that constitute possible continuations of ϕ. The variable p in (14) can be viewed as a hook to which subsequent information is going to be attached. Let us call functions of this type "context change potentials".* We can assume that each sentence is compositionally associated with a context change potential. Thus, for example, the sentence in (15a) would be associated with the context change potential given in (15b) (disregarding tense, for simplicity):

(15) a) He looked tired

b) λp [look tired'$(x) \wedge {}^{\vee}p$].

How can one capture the idea that indefinites set up discourse referents? To illustrate what the main idea is, let us at first put intensionality aside. Let us think of propositions simply as sets of assignments. If ϕ is a formula, let ${}^{\wedge}\phi$ simply be the set of assignments that make ϕ true. We know that propositions ultimately cannot be modelled as sets of assignments. But we are concerned here with those aspects of context-change that have to do with anaphora and all that matters, in this connection, are the kind of anaphoric links that are possible in discourse. The information relevant to this goal can be encoded in sets of assignments, as we shall see. We will want, of course, to modify this view and eventually shift to a more intensional notion of proposition. —

We implement these ideas, by making the following changes to Montague's IL:

(16) Modifications of IL.

(i) we leave out of the language modal and temporal connectives (i.e. '$\Box$', '$\Diamond$', '**P**' and '**F**')

(ii) the interpretation function is only relative to assignments (not to assignments, worlds and instants)

(iii) the set $D_{<s,a>}$ of intensions of type a is defined to be D_a^{Ω}, where Ω is the set of all possible assignments to the variables of IL (i.e. each ω in Ω is a function from $\sqcup_{a \in \text{Type}} \text{Var}_a$ into $\sqcup_{a \in \text{Type}} D_a$.

We assume furthermore the following semantics for the cap '${}^{\wedge}$' and cup'${}^{\vee}$' operators:

(17) a) $[|{}^{\wedge}\alpha|]^{\omega} = \lambda\omega.[|\alpha|]^{\omega}$

b) $[|{}^{\vee}\alpha|]^{\omega} = [|\alpha|]^{\omega}(\omega)$.

With this changes in mind, let us see how the discourse dynamics can be captured. The value of a sentence that contains an indefinite like (18a) can be naturally set to be the context change potential in (18b).

(18) a) A man walked in

* Actually, it might be more appropriate to call context change potential the denotation of '${}^{\wedge}\lambda p$ $[\phi \wedge {}^{\vee}p]$'. I refer to G&S's papers and Chierchia (1982) for discussion.

b) $\lambda p \exists x$ [man'$(x) \wedge$ walk in'$(x) \wedge {}^{\vee}p$].

The variable p in (18b) is inside the scope of $\exists x$. Consequently, continuations of (18a) are going to land inside the scope of $\exists x$. This makes this occurrence of $\exists x$ 'active', i.e. capable of binding pronouns that lie outside of its syntactic discourse. The context change potential in (18b) will characterize the set of assignments described in (19):

(19) The set of all sets of assignments which contain an assignment $\omega[u/x]$ which differs from the current assignment ω at most in that x is mapped into u, where u is a man that walks in.

How is then the sequence constituted by (18a) followed by (15a) going to be interpreted? Since sentences are associated with context change potentials, the natural way to interpret discourse sequencing is by means of function composition. Where A and B are update functions, their composition (which we will denote as '$A \underline{\wedge} B$') is defined as follows:

(20) $A \underline{\wedge} B = \lambda q[A({}^{\wedge}B(q))]$.

So a discourse like the one in (21a) we will be interpreted as in (21b):

(21) a) A man walked in. He looked tired

b) $\lambda p \exists x$ [man'$(x) \wedge$ walk in'$(x) \wedge {}^{\vee}p$] $\underline{\wedge} \lambda p$ [look tired $(x) \wedge {}^{\vee}p$].

By applying the definition of '$\underline{\wedge}$' and working out the reductions, we get:

(22) a) $\lambda q[\lambda p \exists x$[man'$(x) \wedge$ walk in'$(x) \wedge {}^{\vee}p]({}^{\wedge}\lambda p$ [look tired $(x) \wedge {}^{\vee}p](q))]$ def. of '$\underline{\wedge}$'

b) $\lambda q[\lambda p \exists x$[man'$(x) \wedge$ walk in'$(x) \wedge {}^{\vee}p]({}^{\wedge}$ [look tired $(x) \wedge {}^{\vee}q])]$, λ-conv.

c) $\lambda q[\exists x$[man'$(x) \wedge$ walk in'$(x) \wedge {}^{\vee\wedge}$ [look tired $(x) \wedge {}^{\vee}q]]$], λ-conv.

d) $\lambda q \exists x$[man'$(x) \wedge$ walk in'$(x) \wedge$ look tired $(x) \wedge {}^{\vee}q]$, ${}^{\vee\wedge}$-canc.

The result, (22d), is a new context change potential, which can be further composed with subsequent pieces of discourse. In the process of composing the two original context change potentials, the variable x which translates the pronoun *he* gets bound by the quantifier associated with *a man*. The key step is the reduction of (22b) to (22c). This reduction may give the reader the feeling of an improper λ-conversion. But this is not so. The cap-operator abstracts over assignments to variables. In the case at hand, this amounts to abstracting over the variable x in *look tired'(x)*. Thus, that occurrence of x is, in effect bound, which makes the conversion from (22b) to (22c) sound.*

In a parallel fashion, the context change potential associated with (23a) can be set to (23b):

* I refer to Groenendijk and Stokhof (1990) and Chierchia (1992) for details.

(23) a) Every man walked in

b) $\lambda p[\forall x\ [\text{man'}(x) \rightarrow \text{walk in'}(x)] \wedge {}^{\vee}p]$.

Here the variable p is outside of the scope of $\forall x$. This means that subsequent occurrences of x will not be caught by $\forall x$. This quantifier occurrence is not active. The context change potential in (23b) is a "test". It checks whether the current assignment ω satisfies certain conditions. If it does, any proposition containing ω constitutes a possible continuation of (23a). If it doesn't, we get the empty set, which means that we can't update the current context by means of (23a). The difference between (18b) and (23b) corresponds in DRT to the fact that universally quantified NP's introduce "box-splitting" while indefinites do not. Context change potentials as outlined here can be built up compositionally in a Montaguesque way, which gives us a rather neat treatment of donkey-type dependencies, abverbs of quantification and a number of other structures.

The approach we have considered, while promising, needs to be integrated in at least two ways. First, we need a more adequate treatment of propositional content and intensionality. Second, there is a technical problem that must be solved. It is perhaps easier to see the problem if instead of viewing propositions as functions from assignments into truth-values we view them as sets of assignments. An assignments ω assigns a value to each variable, including in particular propositional variables. Normally, ω is viewed as a function i.e. a set of ordered pairs. Since ω also assigns values to propositions, it will contain, among other things, pairs of the form $< p, a >$ where p is a propositional variable and a a set of assignments. Now, if the proposition denoted by p under ω happens to be true at ω, it must be the case that $\omega \in a$. So we must be able to get for some $< p, a >$ that $< p, a > \in \omega$, and $\omega \in a$. But under standard set-theoretic assumptions, which ban loops in $\in$-chains, this is impossible. In other words, the kind of intended semantics for IL runs into the iceberg of the ban on self-application. Any version of Dynamic Intensional Logic that admits dynamic binding over variables of arbitrary type will run into similar problems. Some kind of reflexivity is called for here as well.*

To summarize, I have given some motivation for a theory that has the properties listed in (1a–d). To the best of my knowledge, such a theory has not yet been developed. There are of course many ways to proceed. What I am going to present is, accordingly, not something that is meant to be definitive but rather a progress report. At the same time, I hope to show that a theory with these characteristic constitutes a viable, simple and useful tool for semantic purposes (and, perhaps, for theories of knowledge representation). I will proceed in two stages. I will first develop a theory that has the characteristics in (1a–c), called PT_s (for 'Static Property Theory').** I will then show how to modify it so as to incorporate (1d) and get an account of discourse dynamics that solves the problems mentioned above.

* One of the referees points out that non well-founded set-theory cannot solve the problem for cardinality reasons. Since $|D| \leq |\text{Var} \rightarrow D|$, it follows that $|(\text{Var} \rightarrow D) \rightarrow D| > |D|$, for $|D| > 1$.

** PT_s is a variant of the theory developed (and shown to be consistent) in Chierchia and Turner (1988).

2. The Basic (Static) Theory

The language of PT_s has two categories i (for 'individual') and iu (for 'information unit'). The expressions of category i (in symbol ME_i, for 'meaningful expression of type i') are the terms of the language and the expressions of category iu (in symbols ME_{iu}) are the well formed formulae (wffs). We assume that the language contains a denumerable number of variables and constants of category i (Var_i and Cons_i, respectively). We first recursively define ME_i.

(24) (i) $\mathrm{Var}_i \cup \mathrm{Cons}_i \subseteq \mathrm{ME}_i$

(ii) if $t, t' \in \mathrm{ME}_i$, then $t(t'), \lambda xt, t = t'$ Tt ("it it true that t'', $\neg t$, $[t \wedge t']$, $\forall xt$ are all in ME_i.

Next, we identify the subset of ME_i that constitutes the well-formed formulae (i.e. we define ME_{iu}):

(25) (i) if $t, t' \in \mathrm{ME}_i$, Tt and $t = t'$ are in ME_{iu}

(ii) if $\phi, \psi \in \mathrm{ME}_{iu}$, $\neg\phi$, $[\phi \wedge \psi]$, $\forall x\phi \in \mathrm{ME}_{iu}$.

Some comments are in order. The basis for the language is provided by the untyped λ-calculus. In this language, everything is a term: applications, abstractions, conjunctions of terms, etc. Some terms are also well-formed formulae (i.e. $\mathrm{ME}_{iu} \subseteq \mathrm{ME}_i$). These are the expressions that are going to be evaluated as true or false and are going to display logical behavior. Atomic well-formed formulae have the form "Tt" or "$t = t'$". 'T' can be viewed as a kind of assertion operator or as a truth-predicate. It turns terms into assertions or statements. It can be thought of as akin to natural language predicates like "is true that" or "occurs", although these are more selective than 'T'. We don't care for what happens when we apply 'T' to things that cannot really be asserted or stated (or cannot be said to occur). The language we have allows us to express things like Tx, where x is taken to refer this table. It doesn't matter what truth-value a sentence like this gets (presumably, it will be false).* We only care for what happens when we apply 'T' to proposition-like (or event-like) things. What is to be proposition-like will be specified shortly. Our language is also more liberal than natural language in that it allows to conjoin, negate and quantify over any term whatsoever. Again, it doesn't matter what the conjunction of two entities that cannot really be conjoined amounts to. We only want that conjunction and other operators behave as they should with respect to the well-formed formulae.

The main reason for doing things this way is that we want to make sure to have all the type-freeness and reflexivity we might possibly need. Of course natural language is not quite so tolerant and we will want to reintroduce some type-theoretic

* Bear in mind that Tx, where x is this table is not the translation of the (sortally deviant) natural language sentence "this table is false". The point is that T plays, in part, the role of a truth-predicate, but is more abstract than the natural language predicate "is true". An analysis of "is true" will presumably involve T, but the two predicates should not be immediately identified.

structure for semantic purposes. We shall see that doing so (i.e. introducing in PT_s the amount of typing we need) is fairly straightforward.

I now turn to the semantics for this language. The semantics employs Scott's Domain Theory. In particular, I will adopt here the version of Domain Theory presented in Turner (1990), where all the relevant notions are defined. I will try to present the following material in such a way that also readers not familiar with Scott's approach will be able to follow (if they are willing to take on faith some of the results in Domain Theory).

A PT_s-frame is a structure of the following form:

(26) $< U, [U \rightarrow U], \eta, \partial, IU, O, h >$, where:

- (i) $< U, [U \rightarrow U], \eta, \partial >$ is a Scott model for the λ-calculus. In particular,
  - a) $[U \rightarrow U]$ is the set of continuous functions from U into U*
  - b) η is a continuous isomorphism from $[U \rightarrow U]$ into U
  - c) ∂ is a continuous isomorphism from U into $[U \rightarrow U]$, such that $\eta = \partial^{-1}$.
- (ii) $0 =< f_T, f_\neg, f_\wedge, f_\forall >$ where
  - a) f_T and $f_\neg$ are functions from U into U
  - b) $f_\wedge$ and $f_=$ are functions from $U \times U$ into U
  - c) $f_\forall$ is a function from $[U \rightarrow U]$ into U
- (iii) IU is that subset of U that contains the range of f_T and $f_=$ and is closed under $f_\neg$, $f_\wedge$ and $f_\forall$
- (iv) h maps IU into $\{0, 1\}$. More specifically h is a homomorphism from $< IU, f_\neg, f_\wedge >$ into $< \{0,1\}, \wedge, \neg >$, where $\wedge$ and $\neg$ are the standard truth-functions. Moreover, for any u, $u' \in U$ and $r \in [U \rightarrow U]$, $h(f_=(u, u')) = 1$ iff u identical with u' and $h(f_\forall(r)) = 1 iff h(r(u)) = 1$, for every u.

Building PT_s-frames out of models of the λ-calculus gives us a domain U which is isomorphic under η and ∂ to its own (continuous) function space. Intuitively, η provides us with a representative in U of any function in $[U \rightarrow U]$ and ∂ associates each member of U with a function that corresponds to it. The members of O are logical combinators. They are used to build up information units. The information units are those members of the domain that are either in the range of f_T and $f_=$ or are built out of other information units by means of the logical combinators. Finally, h is essentially a classical evaluation function. It assigns a truth-value to information units respecting their logical structure.

An assignment is a function g from Var_i into U. A model M is a pair $< F, f >$ where F is a PT_s-frame and f a function from Cons_i into U. An interpretation $[|\alpha|]^{M,g}$ relative to a model M and assignment g is recursively defined as follows (I will omit making explicit reference to M).

* Cf. Turner (1990, ch. 2) for the definition of continuity.

(27) a) If $a \in \text{Var}_i$, $[|\alpha|]^g = g(\alpha)$; if $a \in \text{Cons}_i$, $[|\alpha|]^g = f(\alpha)$
b) $[|\lambda xt|]^g = \eta(\lambda u.[|t|]^{g[u/x]})$
c) $[|t(t')|]^g = \partial([|t|]^g)([|t'|]^g)$
d) $[|Tt|]^g = f_T([|t|]^g)$
e) $[|\neg t|]^g = f_\neg([|t|]^g)$
f) $[|t \wedge t'|]^g = f_\wedge([|t|]^g, [|t'|]^g)$
g) $[|t = t'|]^g = f_=([|t|]^g, [|t'|]^g)$
i) $[|\forall x\, t'|]^g = f_\forall(\lambda u[|t|]^{g[u/x]})$.

We say that a formula ϕ is true in M relative to g iff $h([|\phi|]^{M,g}) = 1$.

Our next task is to develop a theory of propositions and properties within this language. Let ‘prop’ (for ‘proposition’) be a distinguished constant of our language and assume that for any term t', ‘prop(t)’ is in ME_{iu}. We will now characterize some structural constraints on ‘prop’. The theory we will develop is based on Frege structures (see Aczel, 1980) and in particular on the version of Frege structures discussed in Turner (1990, ch. 5). The axioms of the theory are the axioms of the λ-calculus, the axioms of first order predicate logic with identity plus a set of axioms on propositions. The axioms on propositions are naturally divided in two lots: (i) axioms stating closure conditions on propositions and (ii) axioms relating propositions to T. The first lot of axiom schemata is as follows:

(28) (i) $\text{prop}(t = t')$
(ii) $\text{prop}(t) \rightarrow \text{prop}(\neg t)$
(iii) $\text{prop}(t) \wedge \text{prop}(t') \rightarrow \text{prop}(t \wedge t')$
(iv) $(\forall x\, \text{prop}(t)) \rightarrow \text{prop}(\forall xt)$.

The second lot of axiom schemata, relating propositions and truth, is as follows:

(29) (i) $Tt \rightarrow \text{prop}(t)$
(ii) $\text{prop}(t) \rightarrow (Tt \rightarrow t)$.

Let us comment on these axioms (which are a simple modification of the those discussed and shown to be consistent in Turner (1990, chs 4 and 5)). The axioms in (28) tell us what propositions are. These axioms are quite straightforward. They establish that identities are propositions and that propositions are closed under the standard connectives and quantifiers (I assume that connectives and quantifiers not mentioned in (28) are defined in the usual way). This imposes a certain amount of structure on propositions. But to have propositions isn't enough. We must be able to assert them. I.e. we must be able to claim that they are true. The assertion operator is what enables us to do so. The axioms in (29) tell us how the assertion operator works. The difference between a proposition and an asserted proposition parallels the natural language distinction between (30a) and (30b) or (31a) and (31b):

(30) a) John run! (what a funny idea!)

b) John runs

(31) a) John's killing Bill

b) John's killing Bill occurred.

'John run' is arguably a clausal structure and carries a certain amount of information. Such information in the form in which it is presented in (30a), however, is not asserted or claimed to be true. It is rather taken as the object of a remark, or as the object of wonder. Per contra, (30b) is asserted. The propositional content merely contemplated in (30a) is claimed to hold in (30b). Similar considerations apply to (31a–b). The 'T' operator is designed partly to capture this distinction.

Let us consider the axioms on 'T'. The first axiom in (29) tells us that only propositions can be true (i.e. can be meaningfully and truthfully asserted). The second axiom tells us that on the set of propositions, the assertion operator behaves like a Tarskian truth-predicate. That is, if t is a proposition, then t and Tt have the same truth-conditions. Consequently for example, if t and t' are propositions, then $T(t \wedge t')$ will be the case iff Tt is the case and Tt' is the case, etc.

There are a couple of observations that can be made in this connection. First, the following is a theorem of PT_s.

(32) a) $\mathrm{prop}(t) \rightarrow Tt \vee T\neg t$

b) proof:

(i)	$\mathrm{prop}(t)$	assumption
(ii)	$\neg(Tt \vee T\neg t)$	assumption
(iii)	$\neg Tt \wedge \neg T\neg t$	from (ii) by elementary logic
(iv)	$\neg T\neg t$	from (iii) by $\wedge$-elimination
(v)	$\neg Tt$	from (iii) by $\wedge$-elimination
(vi)	$\neg t$	from (v) and (i), by axiom (29ii)
(vii)	$\mathrm{prop}(\neg t)$	from (i) by axiom (28ii)
(viii)	$T\neg t$	from (vi) and (vii), by axiom (29ii)

line vii contradicts line iv. Hence by elementary logic (32a) follows.

This theorem shows that propositions behave as they should under T. If something is a proposition, either it or its negation is going to be true. This has a further interesting consequence: the set of information units and the set of propositions do not coincide. In particular, not every information unit (i.e. not every well-formed formula) picks out a proposition. Let us illustrate why this is so. Then we will discuss what type of further consequences this fact may have. Consider the following term:

(33) $\lambda x \neg Tx(x)$.

This expresses a function that applies to things that are not true of themselves. We can thus formulate a version of the liar sentence by applying (33) to itself:

(34) $\lambda x \neg Tx(x)\ (\lambda x \neg Tx(x))$.

Notice the pattern that arises if we perform λ-conversion on (34):

(35) (i) $\neg T\lambda x \neg Tx(x)\ (\lambda x \neg Tx(x))$ λ-conv.
(ii) $\neg T \neg T\lambda x \neg Tx(x)\ (\lambda x \neg Tx(x))$ λ-conv.

If we abbreviate (34) as $R(R)$ ('R' stands for 'Russell Property'), we see that we get a pattern of the form $\neg TR(R)$, $\neg T\neg TR(R)$, $\neg T\neg T\neg TR(R)$ etc. Consider next what happens if we try to claim that $R(R)$ holds:

(36) (i) $TR(R)$, assumption
(ii) prop $(R(R))$, from i., axiom (29i)
(iii) $R(R)$, from i. and ii. by axiom (29ii)
(iv) $\neg TR(R)$, from iii. by λ-conv.

Line iv. contradicts line i., which means that $\neg TR(R)$ is a theorem of PT$_s$. By λ-conversion (cf. (35ii)), $\neg TR(R)$ is equivalent to $\neg T\neg TR(R)$, which is therefore also a theorem of PT$_s$. Now, it is easy to see that $TR(R)$ is an information unit such that neither it nor its negation holds under T. I.e., both $TTR(R)$ and $T\neg TR(R)$ must be false, which means that both $\neg TTR(R)$ and $\neg T\neg TR(R)$ are theorems of PT$_s$. We have already shown that $\neg T\neg TR(R)$ is a theorem. The proof of $\neg TTR(R)$ is a simple variant of (36):

(37) (i) $TTR(R)$, assumption
(ii) prop $(TR(R))$, from i. by axiom (29i)
(iii) $TR(R)$, from i. and ii. by axiom (29ii).

But line ii. is the negation of a theorem of PT$_s$. This shows that assuming $TTR(R)$ leads to a contradiction (just like $TR(R)$ does).

So not every information unit corresponds to a proposition. How are these two notions related? A proposition is a quantity of information that has logical structure (can be conjoined, disjoined, etc.) can be asserted or denied and supports a classical entailment relation. Generally, information units (the denotation of wffs) are propositions. But there are some that are not. The information units that are propositions still have logical structure, can be asserted or denied and support to a certain extent a classical entailment relation. But not fully. Under the truth-predicate, non propositional information units do not behave classically. In a sense, they can be neither asserted nor denied. In other terms, the logic of propositions is classic (this is what theorem (32) shows) and the logic or information units is also classic (for $\phi \vee \neg\phi$ is a tautology and holds for every wff ϕ). But non propositional information units (i.e. information units that correspond to paradoxical sentences) *under* T do not behave classically. And they shouldn't. This is what paradoxicality amounts to in PT_s. While PT_s is based on classical logic, there is in it a touch of non classical logic.

To summarize, the burden of the theory is carried by the axioms in (28) and (29), which appear to be conceptually rather straightforward. PT_s, as currently formulated, is also quite weak. It can be strengthened in various ways. However, our concern here is simply to get a simple logical basis for semantics (a basis that we may have to strengthen as we go along). For these purposes, PT_s suffices. To show this, we indicate how the semantics for a fragment like PTQ can be expressed within PT_s.

3. Montague's PTQ

If we want to reproduce PTQ as is within PT_s, we need to reintroduce type in our theory. This turns out to be easy. We can think of types as a recursively defined family of predicates that carve out into sorts the domain of individuals. Let us begin by defining the set TYPE of types.

(38) (i) e (for "entity") and p (for "proposition") are in TYPE

(ii) if a and b are in TYPE, then $< a, b >$ is in TYPE.

Intuitively, a type of the form $< a, b >$ is the type of functions from a into b. We then add to PT_s the following formation rule:

(39) if $\alpha \in$ TYPE, $t \in \text{ME}_i$, then $\alpha(t) \in \text{ME}_{iu}$.

An information unit of the form "$\alpha(t)$" says that t is of type α.* Types are governed by the following set of axioms:

(40) a) $e(t) \leftrightarrow t = t$

b) $p(t) \leftrightarrow \text{prop}(t)$

c) $< a, b > (f) \leftrightarrow \forall x[a(x) \rightarrow b(f(x))]$.

Axiom (40a) says that every term is of type e. Axiom (40b) says that propositions are of type p. Axiom (40c) says that f is of type $< a, b >$ iff for every x of type a, $f(x)$ is of type b. An illustration of how this system of types works is the following:

(41) a) $< e, p > (f) \leftrightarrow \forall x[e(x) \rightarrow \text{prop}(f(x))]$

b) $< e, < e, p >> (f) \leftrightarrow \forall x[e(x) \rightarrow < e, p > (f(x))]$.

* This means that we must add to the algebra of information units operators corresponding to types. It is trivial to modify PT_s-frames accordingly.

So, f is of type $< e, p >$ (i.e. a 1-place propositional function) just case for every x which is an entity $f(x)$ is a proposition. Similarly, f is of type $< e, < e, p >>$ (i.e. a curried two-place propositional function) just in case for every entity x, $f(x)$ is a one-place propositional function. And so on. Suppose that we assign to *run* type $< e, p >$. This means that we do not care for what the value of *run* is when it applies to something which is not of type e. (We can stipulate, for example, that if x is not of type e, then $\text{run}(x) = \bot$). What we require is that for any x of type e, $\text{run}(x)$ is a proposition.

Having defined the set of types, we can then define quantification and abstraction over entities of type a (a an arbitrary type) as follows:

(42) a) $\forall x_{n,a}\phi \leftrightarrow \forall x_n[a(x_n) \rightarrow \phi]$

b) $\exists x_{n,a}\phi \leftrightarrow \exists x_n[a(x_n) \wedge \phi]$

c) $\lambda x_{n,a}\phi = \lambda x_n[a(x_n) \wedge \phi]$.

This type system corresponds to the one of Cresswell (1973). It doesn't contain functional extensional types and intensional types are not built up by using Montague's phantom type 's'. But for semantic purposes, it works just the same as Montague's type system.

I will now illustrate how our typed version of PT_s can be exploited for semantic purposes by sketching how it can replace Montague's IL in interpreting a PTQ-like fragment of English. Let CN, IV and S be our basic syntactic categories. Complex categories are of the form A/B (where A and B are categories). Each category corresponds to a type. The category-type correspondence is defined as follows.

(43) $\tau : \text{CAT} \rightarrow \text{TYPE}$

a) $\tau(\text{S}) = p$

b) $\tau(\text{CN}) = \tau(\text{IV}) =< e, p >$

c) $\tau(A/B) =< \tau(B), \tau(A) >$

Examples:
$\text{T} = \text{S/CN} \Longrightarrow << e, p >, p >$
$\text{TV} = \text{IV/T} \Longrightarrow <<< e, p >, p >, < e, p >>$
$\text{DET} = \text{T/CN} \Longrightarrow << e, p >, << e, p >, p >>$
$\text{ADV} = \text{IV/IV} \Longrightarrow << e, p >, < e, p >>$.

An expression of category A is interpreted by means of an object of type $\tau(A)$. So for example, a noun phrase (in Montague's notation, an expression of category T) is interpreted as a function from properties into propositions (which corresponds to the type of generalized quantifiers). A transitive verb is interpreted as a function from generalized quantifiers into propositional functions, and so on. I provide next a concrete example of the syntax and semantics of a simple sentence:

(44) every man kisses a unicorn, S, every'(man')(kiss'(a'(unicorn')))

every man, T, every'(man') kiss a unicorn, IV, kiss'(a'(unicorn'))

kiss, TV, kiss' a unicorn, T, a'(unicorn')

a, DET, a' unicorn, CN, unicorn'

Each node in this tree is made up by a triplet constituted by a syntactic expression, its syntactic category and its PT_s- translation. Let us assume that a', every' and kiss' are defined as in (45a–c) respectively:

(45) a) a' = $\lambda P \lambda Q \exists x[P(x) \wedge Q(x)]$
(where P and Q stand for $x_{1,<e,p>}$ and $x_{2,<e,p>}$; x stands for $x_{1,e}$)

b) every' = $\lambda P \lambda Q \forall x[P(x) \rightarrow Q(x)]$

c) kiss' = $\lambda \delta \lambda x \delta(\lambda y[\mathrm{kiss}_*(y)(x)])$
(where δ is $x_{1,<<e,p>,p>}$, kiss_* is of type $< e, < e, p >>$ and x and y are $x_{1,e}$ and $x_{2,e}$).

These definitions are Montague's. In particular, (45c) amounts to Montague's extensionalizing meaning postulate. By applying them, we can reduce the top line of (44) as follows:

(46) a) $\lambda P \lambda Q \forall x[P(x) \rightarrow Q(x)]$ (man')($\lambda \delta \lambda x \delta(\lambda y[\mathrm{kiss}_*(y)(x)])$
($\lambda P \lambda Q \exists x[P(x) \wedge Q(x)]$ (unicorn')))

b) $\forall x$ [man'$(x) \rightarrow \exists y$[unicorn'$(x) \wedge$ kiss$_*(y)(x)$]].

(46b) is the proposition denoted by every'(man')(kiss'(a'(unicorn'))). In PT_s, it is guaranteed to be a proposition, given that man' and *unicorn'* are propositional functions.

This example sketchy though it is suffices to illustrate how easy it is to recast Montague semantics within PT_s. What are the crucial differences with respect to what we had in PTQ? First, in PT_s propositions are not sets of worlds and two logically equivalent propositions will not, in general be equivalent. Second, properties can apply to themselves. This is possible because all the types live in e. Thus, in particular, it follows from our definitions, that everything of type $< e, p >$ is also of type e, which means that if f is of type $< e, p >$, $f(f)$ is a proposition. As a consequence, it is possible to analyze the examples in (47) as shown:

(47) a) John is fun $\Longrightarrow$ fun'(j)
where j is a constant of type e and fun' of type $< e, p >$

b) running is fun $\Longrightarrow$ fun'(run')
where run' is of type $< e, p >$

c) being fun is fun $\Longrightarrow$ fun'(fun').

This simple analysis of the sentences in (47) cannot be obtained within Montague's type system.

There is no need, of course, to adopt a system of types as rich as Montague's (or as rich as the one presented here). We could, for example, adopt as easily more constrained systems of types, such as the one explored in Chierchia (1984). And we could adopt more flexible typings such as those explored in Partee and Rooth (1983). Our theory of propositions is largely neutral as to type theory. I think that this flexibility is an advantage of PT_s. What system of types fits best natural languages is an empirical issue. Consider the theory of simple types that Montague hard wired into his semantics. It plays, essentially, two roles. On the one hand it provides a way to classify semantic domains. On the other hand it provides a paradox free theory of predication: to predicate something of an expression of type a we must go to an higher type. This has as a consequence that we are forced to climb indefinitely up the ladder of types for reasons of attaining sufficient expressability. Suppose we have a function f about which we want to say that q holds. The theory of simple types forces us to classify q as belonging to a type higher than f. But then to say something about q we have to go to a yet higher type. And so on. The present theory claims that predication should be factored out from typing. The question of what semantic domains there are should be kept separate from the question of how to get a paradox-free theory of predication.

There is another point that is worth underlining. While the framework we have set up is more intensional than possible worlds semantics, it also ignores certain fine grained intensional distinctions. So for example, if t is a proposition, then $\lambda x t(a)$ and $t[a/x]$ are the very same object. This is done to facilitate the compositional assignments of meanings to natural language structures (exemplified above in (44)–(46)) and does not entail that there are certain differences in the granularity of natural language meanings that we are bound to be missing. The formulas of PT are not to be immediately identified with natural meanings (i.e. meanings of natural language sentences). Some of them may turn out to pick out natural meanings, but only under specific sets of assumptions concerning the syntactic structures of the relevant portions of natural language and their interpretation. The specific idea embodied in PT is that leaving open the structure of "basic" propositions should suffice to make all of the distinctions that natural language actually makes (and to do so more readily than in possible worlds semantics). Whether this working hypothesis is correct can only be determined by further empirical research into the semantics of natural languages.

4. Dynamicizing Propositions

4.1. PT_d

Our next task is to see what it takes to make our theory more dynamic. As a first step in this direction, we want to reconstruct the approach to discourse dynamics developed by G&S. If we succeed, we will not only have at our disposal in a strongly intensional setting, a treatment of anaphora comparable to the one G&S develop. We will also solve some problems into which the kind of Dynamic Logic developed by G&S runs into (by reaching a more adequate division of labor between context and intensions and by solving the problem with self-reference that having dynamic variables of arbitrary type gives rise to).

The extension of PT_s that meets these desiderata can be obtained by adding a set of dynamic variables to PT_s and by introducing an analogue of Montague's '$^\wedge$' and '$^\vee$' operators that operate on dynamic variables. The technical need for distinguishing two sorts of variables will become clear below. I will call PT_d (for 'Dynamic Property Theory') the result of making these additions to PT_s. I will use the name "discourse markers" to refer to dynamic variables, and when I speak of variables without qualification, both static variables and discourse markers are intended. Let DM be the set of discourse markers. I will use v, u_1, $u_1, \ldots$ for discourse markers, $y, x_1, x_2, \ldots$ for static variables and $\alpha_1, \alpha_2, \ldots$ as metavariables ranging over discourse markers and static variables. The definition of ME_i is thus extended as follows:

(48) (i) $\mathrm{Var}_i \cup \mathrm{DM} \cup \mathrm{Cons}_j \subseteq \mathrm{ME}_i$

(ii) if $t, t' \in \mathrm{ME}_i$ and $\alpha \in \mathrm{Var}_j \cup \mathrm{DM}$, then $t(t')$, $\lambda\alpha t$, $t = t'$, Tt, $\neg t$, $[t \wedge t']$, $\forall\alpha t$, $^\wedge t$, $^\vee t$ are all in ME_i.

The definition of ME_{iu} (the wffs) is the same as before. All (48) does is thus to add a new type of variables and two new terms ($^\wedge t$ and $^\vee t$) to PT_s.

I turn now to characterizing the semantics of PT_d. Static variables are treated as before. Their value is given in terms assignments g: $\mathrm{Var}_i \to U$. We need now also a way to assign a value to discourse markers. Let us call assignments of the latter sort, DM-assignments. The definition of DM-assignments has to be worked out with some care for the following reasons. Intuitively, the cap-operator is going to abstract over (assignments to) discourse markers. So in particular, if ϕ is a formula, $^\wedge\phi$ is going to denote a function from DM-assignments into information units. For example, $^\wedge\mathrm{run}(u_3)$ is going to be a function that maps every DM-assignment c into the information unit that $c(u_3)$ runs. In a sense, $^\wedge\mathrm{run}(u_3)$ is interpreted as a function from environments or contexts into information units. This bears a resemblance to Kaplan's "characters" and to stress it I am going to use Kaplan's term for functions of this type. Now, since characters are going to constitute the denotata of terms of the form '$^\wedge t$', they will have to be part of the domain of our theory. In other words, we must introduce characters into PT-frames. To do so, we exploit Scott's techniques.

What we need is relatively minimal. Let C (for 'contexts') be the set $[N \rightarrow U]$ of all continuous functions from the integers N into U (the domain of our theory). C is, essentially, a set of sequences of elements of U, sequences which are going to play the role of DM-assignments. For C to be continuous, we must assume that N contains a bottom element $\perp_N$ and is ordered as follows:

(49) $1, 2, 3, \ldots, n, \ldots$

$\qquad \perp_N$

The ordering on C is then defined in the standard way, namely:

(50) $c \subseteq c' =_{\text{df}} \forall n \in N(c(n) \subseteq c'(n))$.

Furthermore, for any ω-sequence $< c_i >_{i \in \omega}$ in C of the form $c_1 \subseteq c_2 \subseteq \ldots c_n \ldots$, the least upper bound of the sequence $\sqcup < c_i >_{i \in \omega}$ is also defined standardly as:

(51) $\sqcup < c_i >_{i \in \omega} (n) =_{\text{df}} \sqcup < c_i(n) >_{i \in \omega}$.

If $c \in C$, the value of a discourse marker u_n relative to c is going to be $c(n)$. We will write $c(u_n)$ for $c(n)$. If $c \in C$, then $c[e/u_n]$ (the DM-assignment that differs from c at most in that it assigns e to u_n) is defined as follows:

(52) $c[e/u_n](m) \begin{cases} \perp_U & \text{if } m = \perp_N \\ e & \text{if } m = n \\ c(u_m) & \text{otherwise.} \end{cases}$

So long as all assignments are continuous, it is clear that updated assignments are too (i.e. if $c \in C$, then $c[e/u_n] \in C$).

Having assignments to discourse markers in place, we can introduce characters into PT_s-frames. The set Cr of characters is simply going to be the set of all continuous functions $[C \rightarrow U]$ from C (the set of assignments) into U. So, $Cr = [[N \rightarrow U] \rightarrow U]$. PT_d frames are defined as follows:

(53) A PT_d-frame is a tuple of the form: $< U, [U \rightarrow U] + Cr, \eta, \partial, IU, O, h >$, where:*

(i) a) $[U \rightarrow U]$ is the set of continuous functions from U into U
b) $Cr = [C \rightarrow U]$ (where $C = [N \rightarrow U]$)
c. η is a continuous isomorphism from $[U \rightarrow U] + Cr$ into U
d) ∂ is a continuous isomorphism from U into $[U \rightarrow U] + Cr$ such that $\eta = \partial^{-1}$

(ii) IU, O and h are as in PT_s-frames.

* + is disjoint union. If $D_\perp$ and $D'_{\perp'}$ are Scott domains, $D + D' = \{< 0, d >: d \in D \text{ and } d \neq \perp\} \cup \{< 1, d >: d \in D' \text{ and } d \neq \perp'\} \cup \{\perp\}$. Intuitively, + is a form of union that preserves the ordering of D and D'

A PT_d-model is a pair of the form $< F, f >$, where F is a PT_d-frame and f an assignments to the constants. The interpretation function $[| \quad |]^{M,g,c}$ is now relativized to g and c, where g is an assignment to static variables and c a context (i.e., a DM-assignment). Let $\alpha \in \mathrm{Var}_i \cup \mathrm{DM}$ and β any well-formed expression of PT_d. We adopt the following notational convention:

(54) $$[|\beta|]^{M,g,c}[e/\alpha] = \begin{cases} [|\beta|]^{M,g[e/\alpha],c} & \text{if } \alpha \in \mathrm{Var}_i \\ [|\beta|]^{M,g,c[e/\alpha]} & \text{if } \alpha \in \mathrm{DM}. \end{cases}$$

We now define $[| \quad |]^{M,g,c}$. As usual, we omit making explicit reference to M.

(55) a) If $\alpha \in \mathrm{Var}_i$, $[|\alpha|]^{g,c} = g(\alpha)$; if $\alpha \in \mathrm{DM}$, $[|\alpha|]^{g,c} = c(\alpha)$; if $\alpha \in \mathrm{Cons}_i$, $[|\alpha|]^{g,c} = f(\alpha)$

b) $[|\lambda\alpha t|]^{g,c} = \eta(\lambda e.[|t|]^{g,c}[e/\alpha])$

c) $$[|t(t')|]^{g,c} = \begin{cases} \partial([|t|]^{g,c})([|t'|]^{g,c}) & \text{if } \partial([|t|]^{g,c}) \in [U \to U] \\ \perp & \text{otherwise} \end{cases}$$

d) $[|Tt|]^{g,c} = f_T([|t|]^{g,c})$

e) $[|\neg t|]^{g,c} = f_\neg([|t|]^{g,c})$

f) $[|t \wedge t'|]^{g,c} = f_\wedge([|t|]^{g,c}, [|t'|]^{g,c})$

h) $[|t = t'|]^{g} = f_=([|t|]^{g,c}, [|t'|]^{g,c})$

i) $[|\forall\alpha\, t'|]^{g,c} = f_\forall(\lambda e.[|t|]^{g,c}[e/\alpha])$

j) $[|{}^\wedge t|]^{g,c} = \eta(\lambda c'.[|t|]^{g,c'})$

k) $$[|{}^\vee t|]^{c,g} = \begin{cases} \partial([|t|]^{g,c})(c) & \text{if } \partial([|t|]^{c,g}) \in Cr \\ \perp & \text{otherwise.} \end{cases}$$

Let me illustrate the main novelties here. Clauses (55j–k) provide the semantics for '$^\wedge$' and '$^\vee$'. A term of the form '$^\wedge t$' is going to denote the image in U under η of $\lambda c'.[|t|]^{g,c'}$. $\lambda c'.[|t|]^{g,c'}$ is a function from contexts into the interpretation of t relative to that context. The clause for '$^\vee t$' is set up in such a way as to yield a (non trivial) value just in case it applies to a term that denotes a character. As a consequence, the value of $^{\vee\wedge}\phi$ (where ϕ is a wff) in a context c will be the same as the value of ϕ in c. But the value of, for example, $^\vee\phi$ will be $\perp$. The introduction of a new set of terms induces some changes in the definition of the semantics of application (55c). Their effect is that something of the form $[^\wedge t](t')$ will be undefined.

The interpretation function $[| \quad |]^{g,c}$ is well-defined, provided that for any $t \in \mathrm{ME}_i$, $\lambda c'.[|t|]^{g,c'}$ is in Cr. Since Cr is the set of all continuous function from C into U, in order to show that $\lambda c'.[|t|]^{g,c'}$ is in Cr, we have to show that it is continuous. This can be done by induction on t. In what follows I sketch a proof.

THEOREM: $\lambda c'.[|t|]^{g,c'}$ is continuous.

Proof: in order to show continuity of $\lambda c'.[|t|]^{g,c'}$, we have to show that for any $< c_i >_{i\in\omega}$ in C, $[|t|]^{g,\sqcup < c_i >_{i\in\omega}} = \sqcup < [|t|]^{g,c_i} >_{i\in\omega}$. We give one case, namely when $t = \lambda us$ (s an arbitrary term):

$[|\lambda u\, s|]^{g,\sqcup<c_i>_{i\in\omega}}$ (where $u \in$ DM)

$= \lambda e.[|s|]^{g,\sqcup<c_i>_{i\in\omega}[e/u]}$, semantics of λ

$= \lambda e.[|s|]^{g,\sqcup<c[e/u]_i>_{i\in\omega}}$, def. of $c[e/\alpha]$

$= \lambda e.\sqcup < [|s|]_i^{g,c[e/u]} >_{i\in\omega}$, induction

$= \sqcup(\lambda e. < [|s|]_i^{g,c[e/u]} >_{i\in\omega})$, def. of $\sqcup$ on Cr.

The other cases are similar.

This gives use what we want. The introduction of discourse markers, '$^\wedge$' and '$^\vee$' in PT enables us to recast in it a G&S-style treatment of dynamic anaphoric binding. We sketch how this can be done in the next section.

4.2. Semantics in PT$_d$

The first step is to extend the set TYPE of types so as to have types of the form $< s, a >$:

(56) If a is in TYPE, $< s, a >$ is in TYPE.

Intuitively, if f is of type $< s, a >$, it will be a function from DM-assignments into objects of type a. We will call objects of type $< s, a >$ characters of type a. The term 'intensions of type a' would be inappropriate, for our types are intensional to begin with. Under this typing, we reproduce in PT$_d$ a two-layered semantics, similar to Montague's (and Frege's). But the two layers we obtain are not intensions vs. extensions, but characters vs. intensions.

Types of the form $< s, a >$ are governed by the following principle:

(57) a) $< s, a > (f) \leftrightarrow a(^\vee f)$.

The role of this principle should be intuitively clear. We want characters of type $< s, a >$ to be such that under '$^\vee$' they return to us something of type a.

Notice that this typing automatically gives us discourse markers of any type (because every type lives in e). Moreover, a G&S-style treatment of anaphoric binding becomes straightforward to implement. Let me illustrate by means of an example. The update function corresponding to, say, (58a) will be specified as in (58b) (disregarding tense).

(58) a) A man walked in.

b) $\lambda x_{1,<s,p>} \exists u_{1,e}$ [man'$(u_{1,e}) \wedge$ walk'$(u_{1,e}) \wedge {}^\vee x_{1,<s,p>}$].

We assume that man' and walk' of type $< e, p >$. The variables $x_{1,<s,p>}$ (which in what follows I will abbreviate as 'p') ranges over propositional characters (i.e.

functions from contexts into propositions). Propositional characters (rather than propositions) become the building blocks of the theory. So in the present setting, update functions become functions from propositional characters into propositions. The update function in (58b) can be compositionally associated with (58a) in more than one way, the most straightforward one being an obvious modification of G&S's translation procedure. We also assume that pronouns are translated as discourse markers. For instance, a sentence like (59a) is translated as in (59b):

(59) a) He was wearing a hat

b) λp [wear a hat'$(u_{1,e}) \wedge {}^{\vee}p$].

Discourse sequencing is interpreted as composition, as in G&S:

(60) a) A man walked in. He was wearing a hat

b) $\lambda p \, \exists u_{1,e}$ [man'$(u_{1,e}) \wedge$ walk'$(u_{1,e}) \wedge {}^{\vee}p] \,\underline{\wedge}\, \lambda p$[wear a hat'$(u_{1,e}) \wedge {}^{\vee}p$]

(i) $\lambda q \lambda p \, \exists u_{1,e}$ [man'$(u_{1,e}) \wedge$ walk'$(u_{1,e}) \wedge {}^{\vee}p]$ $({}^{\wedge}\lambda p$[wear a hat'$(u_{1,e})$ $\wedge {}^{\vee}p](q))$, def. of '$\underline{\wedge}$'*

(ii) $\lambda q \lambda p \, \exists u_{1,e}$ [man'$(u_{1,e}) \wedge$ walk'$(u_{1,e}) \wedge {}^{\vee}p]$ $({}^{\wedge}$ [wear a hat'$(u_{1,e})$ $\wedge {}^{\vee}q])$, λ-conv.

(iii) $\lambda q \, \exists u_{1,e}$ [man'$(u_{1,e}) \wedge$ walk'$(u_{1,e}) \wedge {}^{\vee\wedge}$ [wear a hat'$(u_{1,e}) \wedge {}^{\vee}q$]], λ-conv.

(iv) $\lambda q \, \exists u_{1,e}$ [man$(u_{1,e}) \wedge$ walk$(u_{1,e}) \wedge$ wear a hat'$(u_{1,e}) \wedge {}^{\vee}q$], ${}^{\wedge\vee}$-canc.

This gives us a reconstruction of G&S's Dynamic Intensional Logic within PT_d. The main novelty is that we now are in a strongly intensional setting, where discourse markers of any type are available to us. Furthermore, what one works with in PT_d are propositional concepts. This gives us a simple way of dividing up the role of context from the role of intensions. Suppose one tried to add intensions to the dynamic intensional logic sketched in sec. 1.4 by adding worlds to assignments, so that an index is now a pair $< g, w >$ where g is an assignment and w a world.** If '$^{\wedge}$' abstracts over indices, it would simultaneously bind worlds and assignments. Now, in standard Montague Grammar, a sentence like, say, (61a) is analyzed as in (61b):

(61) a) Every man believes that he is a genius

b) $\forall x$ [man'$(x) \rightarrow$ believe'$({}^{\wedge}$genius'$(x), x)$].

* This example makes it also clear, incidentally, why we need both static and dynamic variables. In this formula, 'λq' binds a variable inside the scope of '$^{\wedge}$'. If we only had dynamic variables, they would all be caught by '$^{\wedge}$' and the kind of binding exemplified here (which is part of the definition of dynamic conjunction) would be unexpressable.

** Alternatively one can build into an index information about the denotation of discourse markers and information about the world. This is the option adopted by Groenendijk and Stokhof (1990).

Under the interpretation of '$^\wedge$' just sketched, we would get the wrong results in examples of this kind. The problem is that the occurrence of x in the embedded proposition would not be bound by '$\forall x$'. There are various ways out. For example, one can posit differentiated abstractors '$^{\wedge +}$' and '$^{\wedge \circ}$', where the first operates on the assignment coordinate and the second over the world coordinate of an index. This will lead however to a more complex translation procedure: in updates we will want to use '$^{\wedge +}$', while in embedding '$^{\wedge \circ}$'.

Things appear to work more smoothly in the present set-up. Here '$^\wedge$' is used just in updates functions, where one manipulates contexts. A sentence like (61a) would simply translate as:

(62) $\forall x$ [man'(x) $\rightarrow$ believe'(genius'(x))(x)].

Where *genius(x)* is a proposition.

Considerations such as these are closely related to those that lead Kaplan to separate off characters from intensions and indicate that the present way of exploiting this division of labor is indeed what is called for in dealing with both intra- and inter-sentential binding.

5. Concluding Remarks

The desiderata we have singled out at the outset are all met. PT_d shows that it indeed is possible to have a unified framework where (i) the essential features of Montague Semantics are preserved, (ii) propositions and properties are taken at face value, (iii) properties can apply to themselves and (iv) a compositional approach to certain important types of anaphora can be developed. In particular, we have been able to recast in PT_d Montague Semantics by introducing in it Cresswell's types (which are isomorphic to Montague's intensional types). But there is a lot of room for experimentation in this connection. The domain of our theory is very rich: we are free to explore how to carve it out into subdomains that match the classification schemata and sortal distinctions present in natural languages. In fact, the set up of PT_d appears very useful to study forms of typing more flexible than the standard theory of types.

Terms like "simple" or "natural" are often used as endearing expressions for one's pet formalism. Yet I am willing to take upon myself the risk of indulging in such practice once more and suggest that the present theory is conceptually fairly simple. Its basis is the untyped λ-calculus, which provides us with a well-known and powerful theory of functions. Within this framework we introduce propositions and the means to assert them. All we require is that propositions be closed under logical combinators and support a classical theory of entailment. The structure of properties is determined by the structure of propositions, via the abstraction facilities of the λ-calculus. This approach to propositions has a constructive flavour to it. We impose on them the weakest closure requirements consistent with the pursuit of our goals. What further conditions may be needed can hopefullly be added as we go along.

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